JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 44, No. 4, July–August 2007

Estimating Spare Parts Requirements with Commonality and Redundancy

Robert C. Kline* and Tovey C. Bachman*
LMI, McLean, Virginia 22102

DOI: 10.2514/1.28072

For future human missions to the moon, Mars, or beyond, having the appropriate spare parts may make the difference between mission success and failure. We develop a model for estimating the minimum spare parts mass and volume to meet a required system availability: the percentage of the time the system is not down for lack of parts. The model also produces an optimal resources-vs-system availability curve. Our approach to estimating requirements combines a notional item database with an analytical optimization model. "Notional" items are representative components, the functions of which are those of future items, but the characteristics of which (e.g., reliability, mass) are based on existing items. The basic analytical model starts with item data and mission parameters, such as operating durations, solves stochastic backorder equations, distributes backorders over systems and, thereby, computes system availability as a function of the set of spare parts chosen. We extend the basic model to address both mission phases with several collocated elements that can share spares for common items, and phases with elements that are not collocated and cannot share spares. This extended model enables analysts and spacecraft designers to quickly estimate the benefits of employing common components in combination with other design factors, such as redundancy, criticality, shop-replaceable unit replacement, and resource prioritization. As system designs mature, the model's notional items can be replaced by real items without changing the model or analytical technique. We have implemented the extended model with software in the Spacecraft Sustainability Model$^{\mathrm{TM}}$.

Nomenclature

$A_k(s_1, s_2, \ldots, s_n)$ = availability of an element of type k with spares levels $s_1, s_2, \ldots, s_n$
$\mathrm{EBO}_i(s_i)$ = expected backorders for item type i across all elements, with spares level s_i
$\mathrm{EBO}_{ik}(s_i)$ = expected backorders for item type i across all elements of type k with spares level s_i
i = index of item type i
j = index of subelement type j
k = index of element type k
M_{ik} = number of subelements on elements of type k containing item i
m_i = mass of a unit of item i
N = number of element types
N_k = number of type k elements
n = number of types of items
QPA_{ik} = total number of installed units of item i on a single element of type k
q_{ijk} = number of installed units of item type i on all subelements of type j on a single element of type k
q_{ik} = total number of installed units of item i on all elements of type k
r_i = spares resource of a unit of item i
r_{ik} = share for item i on element k
s_i = total spares level for item type i
s_{ik} = spares level for item type i that can be used only for elements of type k
t_k = operating duration of type k elements
v_i = volume of a unit of item i
α = weighting factor for relative importance of mass and volume
λ_i = mean number of failures of item type i across all elements
λ_{ik} = mean number of failures of item type i on across all elements of type k in operating duration t_k
μ_{ijk} = daily demand rate for a single unit of item type i on subelement type j on element type k

I. Introduction

WE DEVELOP a methodology and a model for estimating mass, volume, and cost of logistics resources (spare parts) for future human missions beyond low Earth orbit (LEO). Because these missions are still in the early stages of development, they require a model that can estimate logistics requirements from early mission-planning data. The same model must be able to refine estimates as the mission architecture matures, by making use of more detailed hardware data as they become available. We first describe a basic model that considers only components that apply to a unique mission element, and then extend the model, permitting a single type of spare part to be used to replace a failed part of that type on any mission element, provided that the elements are collocated (and therefore can share spares). Although we limit ourselves to an operating environment with no resupply, such as would likely be the case with initial Mars surface operations, we note that cases where missions can obtain replenishment spares from nodes of a network in space may become important as the scope of human space exploration expands. We limit ourselves to considering random failures, so we do not capture time-phased removals of components or the possibility that multiple units of a component may have a tendency to all fail at the same time.

The model estimates system performance and required logistics resources using a hybrid parametric-analytic approach. Our performance measure is system availability, which may be thought of as the percentage of time a system is not down for lack of a spare

Presented as Paper 7233 at the Space 2006 Conference & Exhibition, San Jose, CA, 19–21 September 2006; received 29 September 2006; accepted for publication 31 January 2007.

*Research Fellow, Mathematical Modeling Group, 2000 Corporate Ridge. Member AIAA.

part. The hybrid approach combines a notional item database with an analytical (as opposed to simulation) spares optimization model.

"Notional" items are representative components, the functions of which are those of future items, but the characteristics of which (e.g., reliability, mass) are based on those of items either in use on existing hardware, such as the International Space Station or Shuttle, or on items used in laboratory testing. Although the notional item data, furnished by NASA, have been used in reliability analyses for hypothetical Mars and lunar missions, they are unofficial and do not represent any actual planned mission hardware. An analyst may modify the notional item data to fit assumptions about the future items (e.g., reduced mass and increased reliability). Scenario data describe mission phases when a system is operating and the operating durations by phase.

The optimizer uses a proven readiness-based sparing (RBS) methodology to produce an availability-vs-resources curve that can be used to examine the sensitivity of availability to the resource quantity.

We first describe the basic model, using components with unique applications. Next we define two types of commonality, and describe the extended model. We then present the results of several analyses that illustrate the benefits of commonality using our notional data. These analyses use an implementation of the extended model in the Spacecraft Sustainability Model™ (SSM™) software.†

The roots of sparing to an availability target, or RBS, are in the single-echelon, expected backorder minimization models of the late 1950s, developed by mathematical economists Arrow et al. [1] and Scarf [2] at Stanford, and operations researchers Galliher et al. [3] at MIT. By the late 1960s and early 1970s, the Department of Defense was using this approach to set inventory levels for military spare parts, with algorithms developed by Sprung and Hill [4] for the U.S. Air Force, Kohlhass [5] for the U.S. Navy, and Presutti and Trepp [6] for the Defense Logistics Agency. There is now a great variety of single-echelon inventory policies that seek to minimize either expected backorders or backorder occurrences, as described in Silver et al. [7]. As a consequence of Little's law [8], minimizing expected backorders is equivalent to minimizing a customer's wait time for an item.

This approach was extended to optimization of spare parts levels across a supply chain in the 1960s and 1970s by operations researchers such as Sherbrooke [9] and Kruse [10], long before the term "supply chain" existed; the term was then "multi-echelon inventory system." Muckstadt [11] extended this approach to account for the differing effects of backorders for repair parts applying directly to the end item (e.g., aircraft, engine) and those for subassemblies, which do not directly render the end item inoperable. Such models are referred to as "multi-indenture," and many of these models account for repair policies as well as spare parts requirements.

By the late 1970s, models began to focus on maximizing system availability, rather than on minimizing expected backorders; this is RBS. By the early 1980s the U.S. Air Force was using an RBS model that optimized inventory levels across spare parts applying to diverse aircraft types with distinct availability goals, as well as across the supply chain and levels of indenture, as described in O'Malley [12]. The U.S. Army and U.S. Navy began developing their own RBS models at this time. Slay [13] extended this approach to better account for the effect of wholesale backorders' variability on backorders further down the supply chain, and ultimately on aircraft availability. Models to this point treated demand as stationary; trends, due to changing operating tempos, or systems phasing in or out, were not yet accounted for. In the 1980s Hillestadt and Carillo [14], King [15], Kotkin [16], and Slay et al. [17] all developed models that treated dynamic demand patterns. Models began to consider not only the supply chain's impact on readiness, but on the maintenance workload associated with cannibalization as well.

In the 1990s, Kline and Sherbrooke [18] developed an RBS model for spare parts requirements for NASA's Space Station Freedom program, considering periodic resupply of spare parts from Earth. Bachman and Kline [19] described a simpler version of the model presented here, without treatment of common components, in 2004. An earlier version of the commonality discussion we present here appeared in Kline and Bachman [20].

Treatment of common components began at least as early as the 1980s, as documented in [12]. Siddiqi and de Weck [21] have recently developed a model that treats not only common components, but reconfigurable spares and cannibalization of nonoperational redundant units, which we treat here only to a limited extent. A sample of other treatments of commonality may be found in Hofstetter and Wooster [22], Waiss [23], Cronie and Thompson [24], Siegfried [25], Crites and Tremblay [26], Chew and Peng [27], Cheung [28], Gerchak et al. [29], and Baker et al. [30].

II. Basic Model

A. Overview

For simplicity, we first develop a basic model that considers only items with unique applications; we extend the model to handle common items in the section "Extended Model." Our basic model estimates the minimum spares mass and volume required to meet a system availability target, where system availability is the probability that none of the mission elements is inoperable for lack of a spare part. Alternatively, we may seek the spare parts requirement that maximizes availability for a given constraint on spares mass and volume, or cost. The basic model assumes that a spare part, or a redundant installed unit, is required to restore the systems when a component fails; it does not consider cannibalization: removal of a nonoperating spare part of the same type, from another system, to replace a failed component. Although our model can be extended to consider resupply, as described in [17,18], we do not describe that here. The model employs the following six-step process (see Fig. 1):

1) Identify key mission parameters that drive spares mass and volume estimates (mission duration, reliability, environmental factors, etc.).
2) Select major mission hardware from list of notional *elements* (e.g., habitat lander, rover) and systems (e.g., life support, propulsion).
3) Map those elements and systems to notional item data (i.e., parts) that contain failure rates, mass, volume, etc.
4) Adjust the notional item data with failure rate, sizing, and capacity factors to better reflect specific mission hardware.
5) Send the adjusted notional item data and mission parameters (e.g., operating durations) to a spares optimization algorithm.
6) Using that optimization algorithm to estimate the minimum required logistics resources (resource is typically mass, volume, or a convex combination of the two; it may also be cost) to achieve a target system availability, focusing on requirements from random failures of components.

The next several sections of the paper will explain each step in detail.

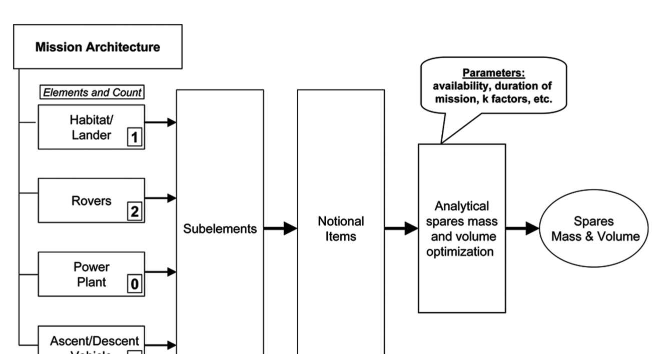

Fig. 1 The steps in requirements process.

†Spacecraft Sustainability Model and SSM are trademarks of the Logistics Management Institute. The SSM is one of the Aircraft Sustainability Model™ (ASM©™) family of sparing models.

When we later move to the extended model that treats commonality, we will modify steps 5 and 6 only, although the notional item data must reflect the fact that there are either components in common between subelements on an element, or components in common across elements. There may be a mixture of items with unique and common applications.

B. Demand Process

Our notional items are line-replaceable units (LRUs), items that may be removed directly from a subelement, as opposed to shop-replaceable units (SRUs), which must be removed from an LRU to repair it. Computing the mean demand rates of items requires notional LRU data, subelement data, and element data. For each notional LRU, we require the following: 1) *element* and *subelement* to which the LRU applies; 2) LRU *failure rate*: the average number of failures per operating hour (inverse of the mean time between failures); 3) *environmental factor*: a failure rate divisor (because each mission phase may have its own environmental factor, this is a default value); 4) the *K-factor*: a failure rate multiplier determined by the type of item: electrical, mechanical, simple mechanical, or structural (not to be confused with the index k for element type); 5) the *duty cycle*: the fraction of the day that an item operates (e.g., a value of 0.5 means the item operates for only half the day); and 6) the *quantity per application (QPA)*: the number of occurrences of the item on a particular system.

LRU mass and volume, although not required for the demand process, are needed to determine resource requirements.

For each subelement, we require the following:

1) A *reliability* factor that adjusts the failure rates of subelement LRUs to reflect components that are more or less reliable than what is reflected in the notional data (the adjusted failure rate is the baseline failure rate multiplied by the reliability factor).

2) A *size* factor that adjusts the unit weight and volume of subelement LRUs for larger or smaller versions of a subelement (e.g., a life-support subelement for a crew of eight might use filters that have twice the weight and volume of filters used for a crew of four). [We treat density (LRU weight per unit LRU volume) as constant.]

3) A *capacity* factor that adjusts the quantity per subelement of each LRU (e.g., a life-support subelement for a crew of eight could have twice as many of each LRU as a life-support subelement for a crew of four).

C. Pipeline

Because we assume no resupply, only spare parts brought on the spacecraft can be used. We further assume that items are not repaired. For an element to be 100% operational for 500 days, each installed item must have enough spares to cover all failures in that period. We treat the number of failures in the operating period as a Poisson random variable, with its mean the product of the operating period and the mean failure rate. This product is often referred to as the mean "pipeline."

We compute an item's pipeline probability density function (PDF) in the simple case when the item is installed on only one type of element, type k (there may be multiple copies of this element, e.g., two rovers) and on only one subelement, type j, of that type of element. In this case, an item's QPA is both the number of units of the item installed on the corresponding subelement and on the element.

In this section we use a simplified version of the notation from the Nomenclature. The total number of units of item type i installed on the subelement of type j on the element of type k is the item's QPA, which is the QPA in the notional data adjusted by the capacity factor for the subelement (of type j) to which the item applies:

$$\mathrm{QPA}_i = (\text{notional QPA})_i \times (\text{capacity factor})_j \tag{1}$$

The mean number of failures per day per installed unit of an item of type i is

$$\mu_i = (\text{failure rate})_i \times (\text{duty cycle})_i \times (K\text{factor})_i \times (24\ \text{hr/day}) \times \frac{(\text{reliability factor})_j}{(\text{environmental factor})_i} \tag{2}$$

and the mean pipeline is

$$\lambda_i = N_k t_k \mathrm{QPA}_i \mu_i \tag{3}$$

For a Poisson process with mean given by Eq. (3), the probability that exactly x units are in the pipeline is

$$p_i(x) = \frac{e^{-\lambda_i}\lambda_i^x}{x!} \tag{4}$$

D. Backorders

Let the number of spares for item type i be s_i. We now compute expected backorders (sometimes referred to as the expected number of "holes": failed units with no replacements). Because the number of backorders with x units in the pipeline is $x - s_i$ for $x > s_i$, and zero for $x \le s_i$, the expected backorders (EBOs) for item i is

$$\mathrm{EBO}_i(s_i) = \sum_{x=s+1}^{\infty} (x - s_i) p_i(x) \tag{5}$$

with $p_i(x)$ from Eq. (4).

E. Availability

Once the EBOs of all items are computed for the range of spares levels s described in the preceding section, the model computes the corresponding system availability for those levels. We treat an item's backorders as uniformly distributed across installed locations, so the probability that a particular location is missing a functioning unit of item i, given a spares level s_i, is $\frac{\mathrm{EBO}_i(s_i)}{q_i}$, where $q_i = N_k \mathrm{QPA}_i$, the total number of installed locations for items of type i across all N_k elements that contain items of type i.

Treating failures of item i in different installed locations as independent, the probability that no randomly selected element of type k is missing an item of type i is

$$A_i(s_i) = \left(1 - \frac{\mathrm{EBO}_i(s_i)}{q_i}\right)^{\mathrm{QPA}_i} \tag{6}$$

The model treats failures of different types of items as independent, so the probability that a randomly selected element of type k is available is simply

$$A(s_1, s_2, \ldots, s_m) = \prod_{i=1}^{n} A_i(s_i) = \prod_{i=1}^{n}\left(1 - \frac{\mathrm{EBO}_i(s_i)}{q_i}\right)^{\mathrm{QPA}_i} \tag{7}$$

From Eq. (7), we can determine the availability for all possible spares levels for all items. This provides the basis for the numerator for the benefit-to-resource ratios we use in marginal analysis.

F. Spares Resources

Resources may be either a combination of mass or volume, or alternatively, cost. When spares mass and volume are the resources, the model adjusts the unit volume and mass of an item installed on a subelement of type j with the subelement scaling parameter size_j as follows:

$$\text{Adjusted Weight (kg)} = \mathrm{size}_j \times \text{weight(kg)}$$

$$\text{Adjusted Volume}(\mathrm{dm}^3) = \mathrm{size}_j \times \text{volume}(\mathrm{m}^3) \times 1000$$

The unit volume is multiplied by 1000, so that the average volume and weight of the items are approximately the same order of magnitude, based on our notional data for a hypothetical Mars mission. This is important when defining the spares resource as a convex combination of mass and volume.

When spares mass and volume are to be minimized, the denominator in the benefit-to-resource ratio is determined by the analyst's decision about the relative importance of minimizing spares mass vs spares volume, conveyed by the choice of the weighting factor α, where $0 \le \alpha \le 1$ and where α is independent of i. Let r_i, m_i, and v_i, respectively, be the spares resource, the mass, and volume of a unit of item i; we set

$$r_i = \alpha m_i + (1 - \alpha) v_i \tag{8}$$

When $\alpha = 1$, the resource is mass alone; we are seeking to find a minimum mass set of spares, and we ignore volume. When $\alpha = 0$ we only consider volume; we seek to find a set of spares that minimizes volume without considering mass. Intermediate values of α are typically desirable, but the proper value of α will depend upon assumptions for a particular mission. If spares cost is to be minimized, the r_i are simply the LRU's unit cost; no weighted resource combination is involved.

G. Optimization

Having determined the availability-to-resource ratios for a wide range of spares levels for each item, we pass those ratios to a marginal analysis algorithm for optimization. A sufficient condition for marginal analysis to produce a unique global maximum of an objective function is that the objective function be a concave function of the independent variables, as described in [12]. A good reference on convex and concave functions is Rockafellar [31]. We will find a suitable objective function and show how the appropriate benefit-to-resource ratios can be computed.

Because there is no resupply, the functions $\mathrm{EBO}_i(s)$ are convex functions of the spares level s, as we see from [17]. Because the negative of a convex function is concave, and because linear combinations of concave functions are concave, the expressions $1 - \frac{\mathrm{EBO}_i(s_i)}{q_i}$ are concave functions of s_i. Furthermore, for a fixed α, s_i is a strictly increasing function of r_i; therefore, these expressions are actually concave functions of r_i.

We reparameterize Eq. (7), obtaining a function of the r_i:

$$A(r_1, r_2, \ldots, r_n) = \prod_{i=1}^{n} A_i(r_i) = \prod_{i=1}^{n} \left(1 - \frac{\mathrm{EBO}_i(r_i)}{q_i}\right)^{\mathrm{QPA}_i} \tag{9}$$

Unfortunately, products of concave functions generally are not concave, so the availability functions in Eqs. (6), (7), and (9) are not concave functions of the spares resources. But logarithm functions are strictly increasing, so the natural logarithm of a concave function is also concave. Furthermore, this strictly increasing property of the log function ensures the logarithm of a function attains its maximum at the same point as the original function. Taking natural logarithms in Eq. (9) yields

$$\begin{aligned} \ln A(r_1, r_2, \ldots, r_n) &= \ln\left\{\prod_{i=1}^{n}\left(1 - \frac{\mathrm{EBO}_i(r_i)}{q_i}\right)^{\mathrm{QPA}_i}\right\} \\ &= \sum_{i=1}^{n} \ln\left(1 - \frac{\mathrm{EBO}_i(r_i)}{q_i}\right)^{\mathrm{QPA}_i} \\ &= \sum_{i=1}^{n} \mathrm{QPA}_i \ln\left(1 - \frac{\mathrm{EBO}_i(r_i)}{q_k}\right) \end{aligned} \tag{10}$$

Because linear combinations of concave functions are concave, the log of availability is a concave function of spares resources, and marginal analysis finds the unique optimal solution for any specified definition of the resource.

In using marginal analysis to find the optimal spares resources to meet an availability goal, we consider the increase in the log of availability from adding one spare for an item of a particular type m, increasing its resource from r_m to r'_m. From Eqs. (6) and (10), this is

$$\begin{aligned} &\ln A(r_1, \ldots, r'_m, \ldots, r_n) - \ln A(r_1, \ldots, r_m, \ldots, r_n) \\ &= \mathrm{QPA}_m \ln\left(1 - \frac{\mathrm{EBO}_m(r'_m)}{q_m}\right) + \sum_{i=1, i\neq m}^{n} \mathrm{QPA}_i \ln\left(1 - \frac{\mathrm{EBO}_i(r_i)}{q_i}\right) \\ &- \mathrm{QPA}_m \ln\left(1 - \frac{\mathrm{EBO}_m(r_m)}{q_m}\right) - \sum_{i=1, i\neq m}^{n} \mathrm{QPA}_i \ln\left(1 - \frac{\mathrm{EBO}_i(r_i)}{q_i}\right) \\ &= \ln A_k(r'_k) - \ln A(r_k) \end{aligned} \tag{11}$$

Thus, the benefit-to-resource ratio for adding one unit of item m is simply

$$\frac{\ln A_k(r'_m) - \ln A(r_m)}{r'_m - r_m} \tag{12}$$

where r_m is the spares resource for spares level s_m, and r'_m is the spares resource for spares level $s_m + 1$.

Starting with no spares, selecting spares step-by-step, and choosing the spare with the largest benefit-to-resource ratio Eq. (12) at each step, we arrive at the unique spares mix that requires the minimum value of the resource [Eq. (8)] to attain our availability goal. This step-by-step process generates the resource-vs-availability curve. Because every point on the curve is optimal, we can use the curve to find the maximum availability attainable for a specified resource constraint, instead of finding the minimum resources to attain an availability target.

III. Extended Model

A. Types of Commonality

We now extend the basic model to handle two types of commonality: *within* an element and *across* elements. For commonality within an element, there are multiple installed units of a component on an element, possibly in different subelements (e.g., navigation, communication), and a single type of spare part suffices to replace any of the failed units, regardless of the subelement on which they are installed. For this type of commonality we further assume that any all elements of a given type are operating together and can share spares. For example, if the mission has two rovers, we assume that they operating together and can share spares, but that a rover and a habitat lander cannot share spares.

For commonality across elements, two or more mission elements (e.g., transit habitat, ascent–descent vehicle) are designed with a common component; that is, each element has a least one installed unit of that component. In this case, if two elements containing a common component are in proximity, a single spare part may be used to replace a failed unit of that part on either element, so that fewer spare parts are required than if each element required a different type of spare part. Elements may have multiple components in common. During those mission phases in which two elements with a common component are not collocated (e.g., a transit habitat in lunar orbit and an ascent–descent vehicle on the lunar surface), failures of that common component must be treated as if there was only commonality within elements.

The two types of commonality may be mixed; that is, we may have commonality within elements for some types of items and commonality across elements for other types of items, but we discuss each type of commonality separately to simplify the discussion.

B. Commonality Within Elements

When we model commonality only within elements, we treat any two items installed on different elements as distinct, so it suffices to consider only elements of a particular type k. As in the basic model, items are LRUs, we assume no resupply, and we refer to the number of demands for an item in its operating duration as the number in pipeline. (In general, the number of units in the pipeline is the number due in from resupply.)

In the case of commonality within elements, the mean number of failures for all items of type i on all N_k elements of type k during the operating duration t_k is

$$\lambda_{ik} = N_k t_k \sum_{j=1}^{M_{ik}} q_{ijk}\mu_{ijk} \tag{13}$$

where the quantities of the item installed and the item demand rates are derived from the notional items by adjustments to reflect the characteristics of a particular mission's hardware, as in the basic model.

For example, suppose we are interested in two rovers (elements of type k) operating together on the surface of Mars. Assume they have a common operating duration of 500 days, and that each rover has a subelement A ($j = 1$) containing two of a given thermal control component of type i, with a failure rate of 0.00002 units/day and a subelement B ($j = 2$), containing three of that same component with a failure rate of 0.000001 units/day. Then the mean pipeline for the thermal control item of type i on rovers would look like

$$\lambda_{i,\text{rovers}} = 2 \times 500 \times [2(0.00002) + 3(0.000001)]$$

The failure rates chosen here are only for illustration; they are not intended to be representative.

Equation (13) is similar to Eq. (3), but because more than one subelement of elements of type k may contain items of type i, we sum across subelements.

As in the basic model, we assume a Poisson PDF for the item pipelines, and the PDF for the pipeline for item i across all elements of type k is

$$p_{ik}(x) = \frac{e^{-\lambda_{ik}}\lambda_{ik}^x}{x!} \tag{14}$$

The particularly simple form of the total pipeline probabilities is due to the properties of the Poisson distribution; in general, computing these probabilities would require convolving the pipeline distributions across subelements.

From this we see the expected backorders for item i across all elements of type k, with spares level s_{ik} is given by

$$\text{EBO}_{ik}(s_{ik}) = \sum_{x=s_{ik}+1}^{\infty} (x - s_{ik}) p_{ik}(x) \tag{15}$$

This generalizes Eq. (5), the difference being that there may be multiple element types (different values of k), each with its own spares and expected backorders for items of type i.

Note that the number of items of type i installed on a single subelement of type j on an element of type k is q_{ijk}, the total (across subelements) number of items of type i installed on an element of type k is QPA_{ik}, there are N_k elements of type k, and the total installed locations for item i across all elements of type k is q_{ik}, so that we have

$$\text{QPA}_{ik} = \sum_{j=1}^{M_{ik}} q_{ijk} \quad \text{and} \quad q_{ik} = N_k \text{QPA}_{ik} \tag{16}$$

In our example with the rovers, we have

$$\text{QPA}_{i,\text{rover}} = 2 + 3 = 5 \quad \text{and} \quad q_{i,\text{rovers}} = 2 \times 5 = 10$$

Treating backorders of item i as uniformly distributed across installed locations on elements of type k, the availability of an arbitrary element of type k due to item i is

$$A_{ik}(s_{ik}) = \left(1 - \frac{\text{EBO}_{ik}(s_{ik})}{q_{ik}}\right)^{\text{QPA}_{ik}} \tag{17}$$

Considering all types of items on elements of type k, and treating failure of distinct types of items as independent, we find the availability of an arbitrary element of type k is

$$A_k(s_{1k}, s_{2k}, \ldots, s_{nk}) = \prod_{i=1}^{n} A_{ik}(s_{ik}) = \prod_{i=1}^{n}\left(1 - \frac{\text{EBO}_{ik}(s_{ik})}{q_{ik}}\right)^{\text{QPA}_{ik}} \tag{18}$$

We may think of this availability as the expected (i.e., mean) fraction of the elements of type k not down for lack of a part, or alternatively, the probability that a randomly chosen element of type k is not down for lack of a part. This generalizes the result in the section "Basic Model" by allowing for demand rates and installed quantities of a given type of item to vary over the subelements on which the item is installed.

C. Commonality Across Elements

We now consider the case in which multiple types of elements share common items, and a mission phase in which all the elements we are considering are collocated, so that they can share spare parts for common items. We assume that all elements of a given type k either operate or do not operate during a mission phase, and that an item of type i is either operating on all elements of a type k during a mission phase, or operating on none of those elements of type k during that phase. Although this is less general than the approach found in [21], it also permits a somewhat simpler treatment of commonality. Let $\delta_{ik} = 1$, if all items of type i are operating on elements of type k during the mission phase under consideration; and $\delta_{ik} = 0$ if no items of type i are operating on elements of type k during the mission phase under consideration.

Let λ_{ik} be defined as in Eq. (13).

Then the mean number of demands for item i in the mission phase under consideration is

$$\lambda_i = \sum_{k=1}^{N} \lambda_{ik}\delta_{ik} = \sum_{k=1}^{N} \delta_{ik}\left(N_k t_k \sum_{j=1}^{M_{ik}} q_{ijk}\mu_{ijk}\right) \tag{19}$$

and the Poisson distribution for the number of units of item i in the total (due to all elements) pipeline is

$$p_i(x) = \frac{e^{-\lambda_i}\lambda_i^x}{x!} \tag{20}$$

Again we use properties of the Poisson distribution to move from Eq. (19) to Eq. (20) without the effort of explicitly computing convolutions of distributions. From Eq. (20), we develop expressions for total expected backorders due to item i as before:

$$\text{EBO}_i(s_i) = \sum_{x=s_i+1}^{\infty} (x - s_i) p_i(x) \tag{21}$$

In contrast to Eq. (15), where each item type i had a separate spares level and separate backorders for each element type to which it applied, in Eq. (21) we pool all of our backorders (holes) and spares for a given item type. The ability to use the total spares level s_i to fill any of the holes across elements amounts to computing a common safety level, because a spares level is expected leadtime demand plus safety level [7,17]. Experience has shown that this typically results in fewer spares than we would have by summing the spares levels s_{ik} across element types. Although we suspect that one can develop sufficient conditions under which this can be proven rigorously, we have not done so here. We illustrate the typical savings in spares in the next section.

To estimate the effect of these backorders on each element, we assign backorders to each element in proportion to its share of the total pipeline, as follows:

$$\text{EBO}_{ik}(s_i) = \frac{\lambda_{ik}}{\lambda_i}\text{EBO}_i(s_i) = \frac{\lambda_{ik}}{\lambda_i}\sum_{x=s_i+1}^{\infty}(x - s_i) p_i(x) \tag{22}$$

We now compute the availability of elements of type k due to element i as

$$A_{ik}(s_i) = \left(1 - \frac{\text{EBO}_{ik}(s_i)}{q_{ik}}\right)^{\text{QPA}_{ik}} \tag{23}$$

Note that this is similar to Eq. (17) but with a spares level that does not depend on the element type because of the proration of backorders.

The availability of an arbitrary element of type k, due to all types of items, is given by

$$A_k(s_1, s_2, \ldots, s_n) = \prod_{i=1}^{n} A_{ik}(s_i) = \prod_{i=1}^{n}\left(1 - \frac{\text{EBO}_{ik}(s_i)}{q_{ik}}\right)^{\text{QPA}_{ik}} \tag{24}$$

Once the optimal set of spares is found, as in the basic model, the total number of spares for a given item is divided between elements with each element receiving spares in proportion to its share of the total mean pipeline:

$$s_{ik} = \frac{\lambda_{ik}}{\lambda_i} s_i = r_{ik} s_i \tag{25}$$

where r_{ik} is the share for item i on element k.

D. Example

Although we have not proven it, commonality can lead to significant reductions in spares requirements for a given performance goal. We illustrate this with an example. For simplicity, the example does not use the availability-based optimization, but in practice we observe benefits with that optimization similar to those we show here. Suppose a common LRU is installed on two different elements. Element A contains that item with a mean pipeline of nine (i.e., the average failures over the mission equals nine). Element B contains the same type of item with a mean pipeline of 16. Throughout the example we assume pipeline PDFs are Poisson, so one standard deviation of the pipeline equals the square root of the mean pipeline. For simplicity, we also assume the spares requirement is a safety level of two standard deviations plus the mean pipeline [i.e., a probability of sufficiency (POS) of 97.7%], rather than using a requirement driven by an availability goal.

If we establish spares levels for items separately, and do not share spares, the spares requirement for element A is $9 + 2\sqrt{9} = 15$, and the requirement for the item in element B is $16 + 2\sqrt{16} = 24$. Hence, the total spares level requirement is 39 (see Table 1). The total EBOs with 39 spares is 0.094. (The model uses EBOs to compute availability as we saw in Eqs. (18) and (24).

But by building a single safety level for the common LRUs, and sharing spares across elements, we can do better: The total spares level is $9 + 16 + 2\sqrt{9 + 16} = 35$. Therefore, considering commonality reduces the spares requirement by four spares (see Table 1). Furthermore, the EBOs drop from 0.094 to 0.060 (see row 4 of Table 1).

As we saw in Eqs. (24) and (25), the model considers the pipelines and backorders from all common items simultaneously, and assigns the items on each element a prorated share of the backorders and spares. In Table 1, the *share* factor [Eq. (25)] for each item is the ratio of the individual mean pipeline to the total mean pipeline. So for the common item on element A, the share is 0.36 (i.e., 9 ÷ 25), or 36% of the total mean pipeline. Thus, the item on element A receives 36% of the 0.060 EBOs. From Eq. (24) we know these reductions in EBOs equate to improved availability as well.

Table 1 Benefits of commonality

Basis of spares levels	Mean pipeline	Safety stock[a]	Spares requirement	EBOs	Shares
Element A	9	6	15	0.043	0.36
Element B	16	8	24	0.051	0.64
Without commonality	25	14	39	0.094	——
With commonality	25	10	35	0.060	——
Savings for commonality			4		

[a]Safety stock is two standard deviations, or two times the square root of the mean pipeline.

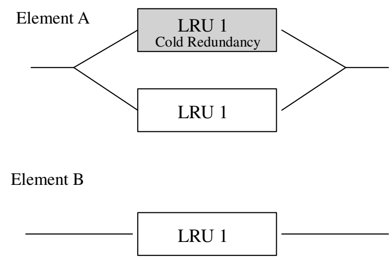

Fig. 2 Redundancy serves as spare.

E. Redundancy and Commonality

The model's treatment of commonality affects the way it considers redundancy. The item data specifies which items are redundant and which are not. In treating redundancy, the model assumes an item with a redundant installed unit can withstand a single failure and still operate. Thus for redundancy, the model imparts a benefit of one "free spare" for that item (no resource expenditure). This assumes the redundant item is in a cold standby state (not operating) and, if needed, can serve as a spare for any installed unit of a common item. This is the case in Fig. 2, where the redundant LRU in element A can serve as a spare to the operating unit of the same item in element B.

If several installed units of a common item are redundant, the model sums those "free" spares across all redundant items. In the optimization, the model examines the sum of the pipelines and the sum of the redundant spares to determine if additional spares are required.

At this time we have not resolved whether we should extend the model further to 1) treat multiple levels of redundancy for a particular item, and 2) treat the difference between the ability to resolve failures with redundancy vs with a spare.

IV. Analyses

A. Spacecraft Sustainability Model

This section describes analyses performed with the SSM software [32] that implements the methodology described in the section "Extended Model." We demonstrate how the use of common components can significantly reduce spares weight, and illustrate the sensitivity of spares weight to the percentage of common components present.

The SSM analyses use the notional item data discussed in the Introduction: the analyst adjusts these data to reflect estimated future hardware characteristics. The SSM produces not only an estimate of spares requirements, but a resource-vs-system availability curve as discussed in the section "Basic Model." The curve enables the analyst to rapidly judge the sensitivity of performance and resource requirements to reliability improvements, miniaturization, or changes in a variety of mission factors. As system designs mature, real items can replace notional items without changing the model or analytical technique.

All analyses that follow use the notional data described earlier, assume a 500 day mission, and a 95% availability target. The notional data describe a transit habitat, habitat lander, an ascent–descent vehicle, in situ resource utilization and power plants, and rovers considered for a Mars mission similar to those discussed in Hoffman and Kaplan [33]. Thus the total element mass is substantially more than the mass appropriate for short-duration lunar missions now being developed, and the spares weights in our results reflect that (the data could be adjusted to reflect smaller mission elements appropriate to an initial lunar mission, but we have not done so here). Furthermore the analyses do not consider redundancy, reduced spares for less critical items, or other considerations that also provide weight and volume reduction, so they demonstrate the impacts of commonality alone.

B. Impact of Commonality

Increasing the degree of commonality of components across elements can significantly reduce spares mass and volume. We

Table 2 Effect of commonality on habitat lander and ascent–descent elements' spares weight (95% availability target)

Analysis	Weight	Volume
Commonality within elements	10,727 kg	32, 639 dm^3
Commonality across elements	9,530 kg	29, 173 dm^3

demonstrate this by comparing the spares weight and volume required for treating commonality only within elements to that required for commonality across elements, using habitat lander and ascent–descent elements that have about 30% of their parts in common. Table 2 shows that moving from commonality within to commonality across elements results in a 10% spares weight reduction. Changing the treatment of commonality from within to across elements in the SSM only requires changing a single model setting and no changes to the data.

Figure 3 shows this same comparison between commonality within and across elements, but does so for all points on the availability-vs-resources curve produced by the SSM spares optimization. The curve is developed using the benefit cost ratios described in Eq. (12). Notice that the benefits of commonality are sensitive to the availability and increase as availability increases except near the top of the curve. From this curve an analyst immediately sees the increase in resources necessary to move to a higher performance level or, alternatively, the decrease in performance arising from constraining resources.

C. Impact of Percentage of Common Items

To illustrate the sensitivity of spares weight to the percentage of common items in the overall item population, we examine three different data sets, each with different percentages of common items. (These data sets are for demonstration purposes only because results may vary depending on data and assumptions.)

The three data sets correspond to the rows in Table 3. The first row shows results for the habitat lander and ascent–descent elements from Table 2. The next row uses those elements plus other elements from Sec. IV.A. The final row shows a "best-case scenario" for commonality where we have two identical elements or 100% commonality. In Table 3, the percentage of common items shown in column 2 is the number of common items shown in column 5 divided by the number of items shown in column 4. Column 3 shows the percentage reduction in spares weight between the results in columns 6 and 7.

Table 3 shows that the spares weight reduction from moving from commonality within to commonality across elements may be as low as 11% when only one-third of the items are common, as is the case with the collocated habitat lander and the ascent–descent vehicle (row 1). It may be as high as a 33% when all items are common between two elements (row 3). These results demonstrate that the percentage of common items can have a significant effect on the magnitude of spares weight reduction, and underscore the importance of properly considering commonality in the notional data. The results also have implications for spares requirements in mission phases in which elements are not collocated (elements cannot share spares: higher requirements) compared to phases when they are collocated (can share spares: lower requirements). Again, we emphasize that these results do not give a precise estimate of the magnitude of spares weight reduction for any specific mission; they only show that the treatment of commonality can make a significant difference in required spares weight.

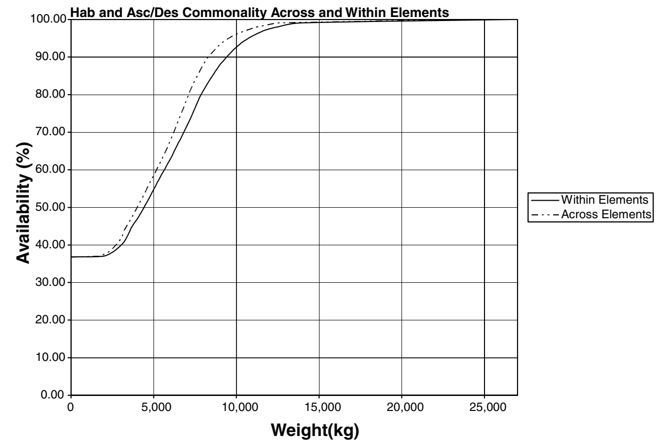

Fig. 3 Habitat lander and ascent–descent vehicle availability curve.

V. Conclusions

We have developed a hybrid parametric-analytic model for estimating required spares mass and volume for future human missions beyond low Earth orbit, early in the design process. We have implemented the model in software known as the Spacecraft Sustainability Model^{TM}. The model assists spacecraft designers and mission planners who want to know, from a spare parts perspective, if a design is feasible, or which design is preferable. It also helps engineers and systems designers by pinpointing systems in which reliability improvements or miniaturization would be most effective at reducing spares mass and volume before definite hardware descriptions are available. Data requirements are limited (owing to the use of notional data to represent mission hardware) and the model performs assessments or sensitivity analysis in just a few minutes.

We have extended the model to consider components with common applications on multiple mission elements. By using common components across multiple elements, the mission can plan on one type of spare part instead of the two different parts that would be required by designs that do not exploit commonality. The result is lower spares mass and volume (resources), or improved system availability for the same spares resources.

The enhanced model enables the analyst to quantify, at least approximately, the savings in spares mass and volume from employing common components in hardware designs, and to perform sensitivity analyses on this aspect of hardware design.

There are other issues related to commonality that are not addressed here, such as reduced complexity of inventory tracking and storage, simplification of maintenance procedures, and the like. Our analysis also does not consider the resource savings from using a less critical common part as a spare to replace a failed part with a more critical application, even though the SSM can analyze just such a scenario.

Our next steps include improving modeling of redundancy and applying the SSM to estimate spares requirements for a crew exploration vehicle NASA is examining.

Table 3 Weight reduction with commonality (95% availability target)

Data set	Percentage		Number of items		Weight	
1-Input data	2-Common items	3-Reduction in weight	4-Total	5-Common	6-Commonality within elements	7-Commonality across elements
Habitat lander+ ascent–descent vehicle	29%	11%	448	130	10,727	9,530
All notional data	81%	31%	878	714	25,114	17,427
Two habitat landers	100%	33%	602	602	14,160	9,529

References

[1] Arrow, K, J., Karlin, S., and Scarf, H., *Studies in the Mathematical Theory of Inventory and Production*, Stanford Univ. Press, Stanford, CA, 1958.

[2] Scarf, H., "The Optimality of (s, S) Policies in the Dynamic Inventory Problem," *Mathematical Methods In the Social Sciences*, Stanford Univ. Press, Stanford, CA, 1960, Chap. 13.

[3] Galliher, H. P., Morse, P. M., and Simond, M., "Dynamics of Two Classes of Continuous Review Inventory Systems," *Operations Research*, Vol. 7, No. 3, 1959, pp. 362–384.

[4] Sprung, J., and Hill, J., *Shadow Price Investigation of the Marginal Analysis OFM Requirements Algorithm*, Headquarters, Air Force Logistics Command, Wright–Patterson Air Force Base, OH, 1970.

[5] Kohlhaas, P. C., "Variable Operating and Safety Level (VOSL) Requisitions Short Model," Navy Fleet Material Support Office, ALRAND Working Memorandum 144, 1968.

[6] Presutti, V. J., and Trepp, R. C., "More Ado About Economic Order Quantities (EOQ)," *Naval Research Logistics Quarterly*, Vol. 17, No. 2, 1970, pp. 243–252.

[7] Silver, E. A., Pyke, D. F., and Peterson, R., *Inventory Management and Production Planning and Scheduling*, Wiley, New York, 1998.

[8] Little, J. D. C., "A Proof of the Queuing Formula: $L = \lambda W$," *Operations Research*, Vol. 9, No. 3, May 1961, pp. 383–387.

[9] Sherbrooke, C. C., "METRIC: A Multi-Echelon Technique for Recoverable Item Control," RAND, Memorandum RM-5078-PR, 1966.

[10] Kruse, W. K., "An Exact N-Echelon Inventory Model: The Simple Simon Method," U.S. Army Inventory Research Office, Technical Report 79-2, 1979.

[11] Muckstadt, J. A., "A Model for a Multi-Item, Multi-Echelon, Multi-Indenture Inventory System," *Management Science*, Vol. 20, No. 4, 1973, pp. 472–481.

[12] O'Malley, T. J., *The Aircraft Availability Model: Conceptual Framework and Mathematics*, LMI, McLean, VA, 1983.

[13] Slay, F. M., "VARI-METRIC, An Approach to Modeling Multi-Echelon Resupply when the Demand Process is Poisson with a Gamma Prior," LMI, Working Note AF301-3, 1984.

[14] Hillestad, R. J., and Carrillo, M. J., "Models and Techniques for Recoverable Item Stockage when Demand and the Repair Process are Non-Stationary—Part 1: Performance Measurement," RAND, Note N-1482-AF, 1980.

[15] King, R. M., "Assessing Aircraft Spares Support in a Dynamic Environment," LMI, Working Note AF401-3, 1985.

[16] Kotkin, M. H., "Operating Policies for Non-Stationary Two-Echelon Inventory Systems for Reparable Items," U.S. Army Material Systems Analysis Activity, Special Publication, No. 39, 1986.

[17] Slay, F. M., Bachman, T. C., Kline, R. C., O'Malley, T. J., Eichorn, F. L., and King, R. M., "Optimizing Spares Support: The Aircraft Sustainability Model," LMI AF501MR1, McLean, VA, Oct. 1996.

[18] Kline, R. C., and Sherbrooke, C. C., "Estimating Spares Requirements for Space Station Freedom Using the M-SPARE Model," LMI NS101R2, McLean, VA, July 1993.

[19] Bachman, T. C., and Kline, R. C., "Model for Estimating Spare Parts Requirements for Future Missions," AIAA Paper 2004-5978, 2004.

[20] Kline, R. C., and Bachman, T. C., "Estimating Spare Parts Requirements with Commonality for Human Space Missions," AIAA Paper 2006-7233, 2006.

[21] Siddiqi, A., and de Weck, O. L., "Spare Parts Requirements for Space Missions with Reconfigurability and Commonality," *Journal of Spacecraft and Rockets* (to be published).

[22] Hofstetter, W., Wooster, P., Nadir, W., and Crawley, E., "Affordable Human Moon and Mars Exploration Through Hardware Commonality," AIAA Paper 2005-6757, Aug. 2005.

[23] Waiss, R. D., "Cost Reduction on Large Space Systems Through Commonality," AIAA Paper 1987-585, Jan. 1987.

[24] Cronie, M. A., and Thompson, P. R., The Economics of Commonality —A Boeing Product Strategy View," AIAA Paper 1980-1839, Aug. 1980.

[25] Siegfried, W. H., "Application of Commonality Criteria to the Design of Lunar and Mars Equipment," AIAA Paper 1993-4216, Sept. 1993.

[26] Crites, J. A., and Tremblay, P. G., "Space Station Maintainability Design Requirements for Life Cycle Costs (Commonality and Standardization)," AIAA Paper 1988-4749, Oct. 1988.

[27] Chew, E. K., and Peng, L., "Component Commonality in Assembled-to-Stock Systems.," *IIE Transactions*, Vol. 38, No. 3, March 2006, pp. 239–251.

[28] Cheung, K. L., "The Effects of Component Commonality in an Infinite Horizon Inventory Model," *Production Planning and Control*, Vol. 13, No. 3, April 2002, pp. 326–333.

[29] Gerchak, Y., Magazine, M. J., and Gamble, B., "Component Commonality with Service Level Requirements," *Management Science*, Vol. 34, No. 6, June 1988, pp. 753–760.

[30] Baker, K. R., Magazine, M. J., and Nuttle, H. L., "The Effect of Commonality on Safety Stock in a Simple Inventory Model.," *Management Science*, Vol. 32, No. 8, Aug. 1986, pp. 982–988.

[31] Rockafellar, R. T., *Convex Analysis*, Princeton Univ. Press, Princeton, NJ, 1970.

[32] Kline, R. C., Bachman, T. C., and DeZwarte, C. A., "LMI Model for Logistics Assessments of New Space Systems and Upgrades," LMI NS520T1, McLean, VA, 2006.

[33] Hoffman, S. J., and Kaplan, D. I., *Human Exploration of Mars: The Reference Mission of the NASA Mars Exploration Study Team*, NASA, Special Publication 6107, 1997.

J. Korte
Associate Editor